OPETUSHALLITUS
UTBILDNINGSSTYRELSEN

OR = Operations or Operational Research

relation with healthcare

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What is Operations Research?

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Operations Research

- Today, the term operations research means a scientific approach to decision making, which seeks to determine how best to design and operate a system, usually under conditions requiring the allocation of scarce resources.
- During World War II, British military leaders asked scientists and engineers to analyze several military problems…

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Operations Research

1947 Simplex Method for Linear Programming

G. DANTZIG

1950s Network Flows

FORD-FULKERSON

1957 Dynamic Programming

R. BELLMAN

1960s Integer Programming

GOMORY, LAND-DOIG, DAKIN

Large Scale LPs

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Operations Research

1970s Combinatorics ; Computational Complexity

1979 Ellipsoid Method for LP KHAGIAN

1983 Projection Method for LP KARMARKAR

1990s Development of heuristics
Improvement of exact methods
Proliferation of applications

NOW Meta-heuristics
Hybrid methods

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“I believe that the greatest impact of the quantitative approach will not be in the area of problem solving, although it will have growing usefulness there.

Its greatest impact will be on problem formulation: the way managers think about their problems…”

Robert. H. Hayes

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Real creativity
is stronger
than the narrow
approach of the
“O.R.-mathematician”

Ideal:
creativity
+ O.R. methodology

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Some examples

- Blending problems
- Land evaluation
- Waste collection
- Distribution and location
- Investment analysis
- Planning

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An example of linear programming

- Giapetto’s Woodcarving, Inc.

- Production of wooden toys
  - Soldiers
  - Trains

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Available data

	soldier	train
selling price/unit	€27	€21
raw material price/unit	€10	€9
production cost/unit	€14	€10
finishing labor/unit	2 h	1 h
carpentry labor/unit	1 h	1 h
market restriction	40	∞
per week : 100 finishing hours, 80 carpentry hours		

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Decision variables

- Decision of Giapetto?

  determining production quantities

- X_1 = number of soldiers to produce each week
- X_2 = number of trains to produce each week

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Objective function

- Goal of Giapetto?

  turnover maximization? cost minimization? profit maximization?

  profit = 27 –10 – 14 = €3 per soldier

  21 – 9 – 10 = €2 per train

  max $3\ X_1 + 2\ X_2$

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Constraints

X_1 and X_2 must satisfy a number of constraints

- no more than 100 h of finishing time

$$2 X_1 + X_2 \leq 100$$

- no more than 80 h of carpentry time

$$X_1 + X_2 \leq 80$$

- at most 40 soldiers should be produced

$$X_1 \leq 40$$

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- Additional constraints:

  X_1 en $X_2 \geq 0$

- Objective function coefficients / technological coefficients

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Complete model

- LP Model :

$$\max z = 3X_1 + 2X_2 \qquad (1)$$

s.t.

$$
\begin{aligned}
2X_1 + X_2 &\leq 100 && (2)\\
X_1 + X_2 &\leq 80 && (3)\\
X_1 &\leq 40 && (4)\\
X_1 &\geq 0 && (5)\\
X_2 &\geq 0 && (6)
\end{aligned}
$$

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Linear means what?

- Proportionality
- Additivity
- Divisibility
- Certainty

- Not all conditions are always satisfied: simplications or relaxations required

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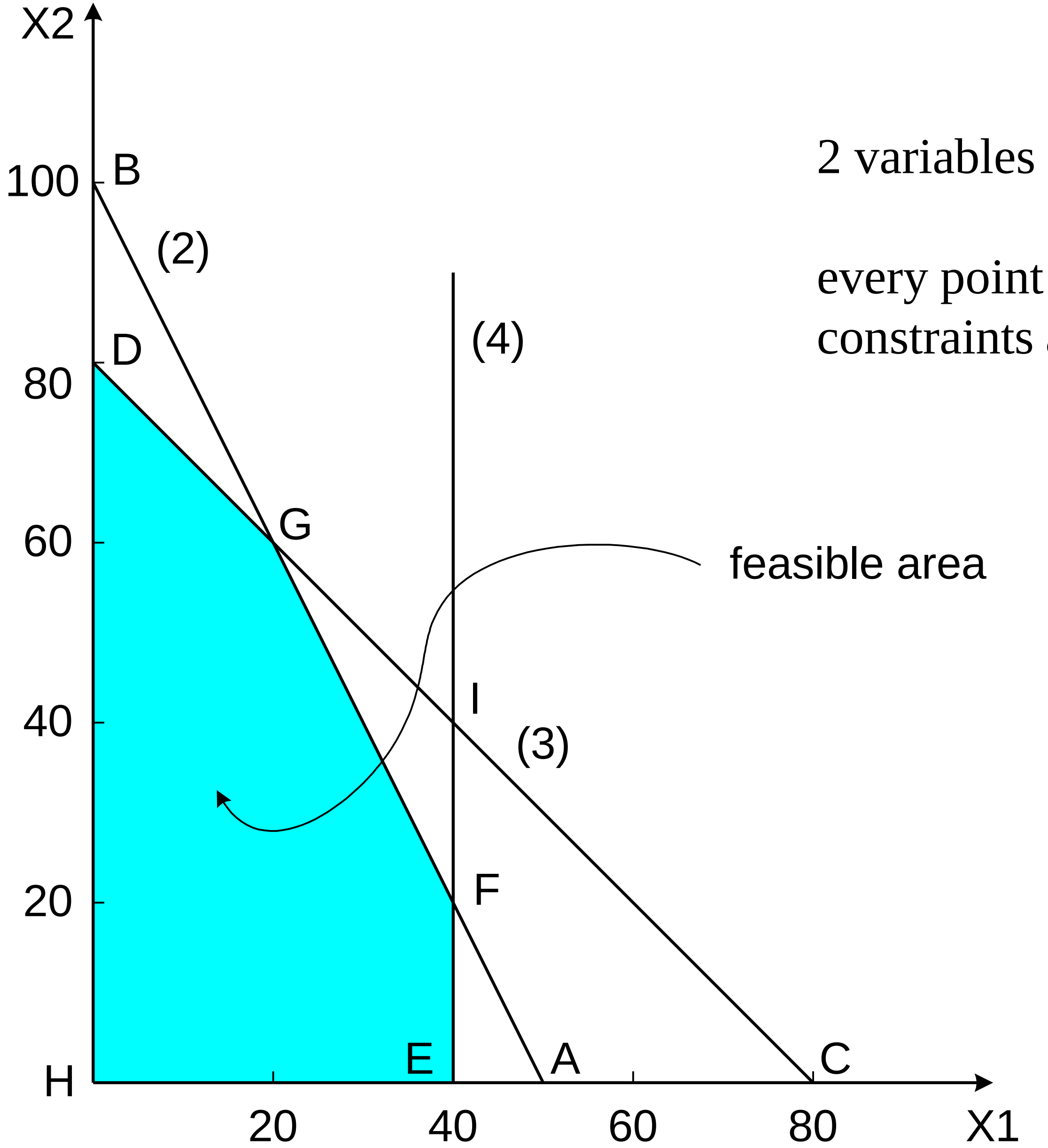

2 variables → graphical representation

every point in the feasible area satisfies the different constraints and represents a feasible production plan

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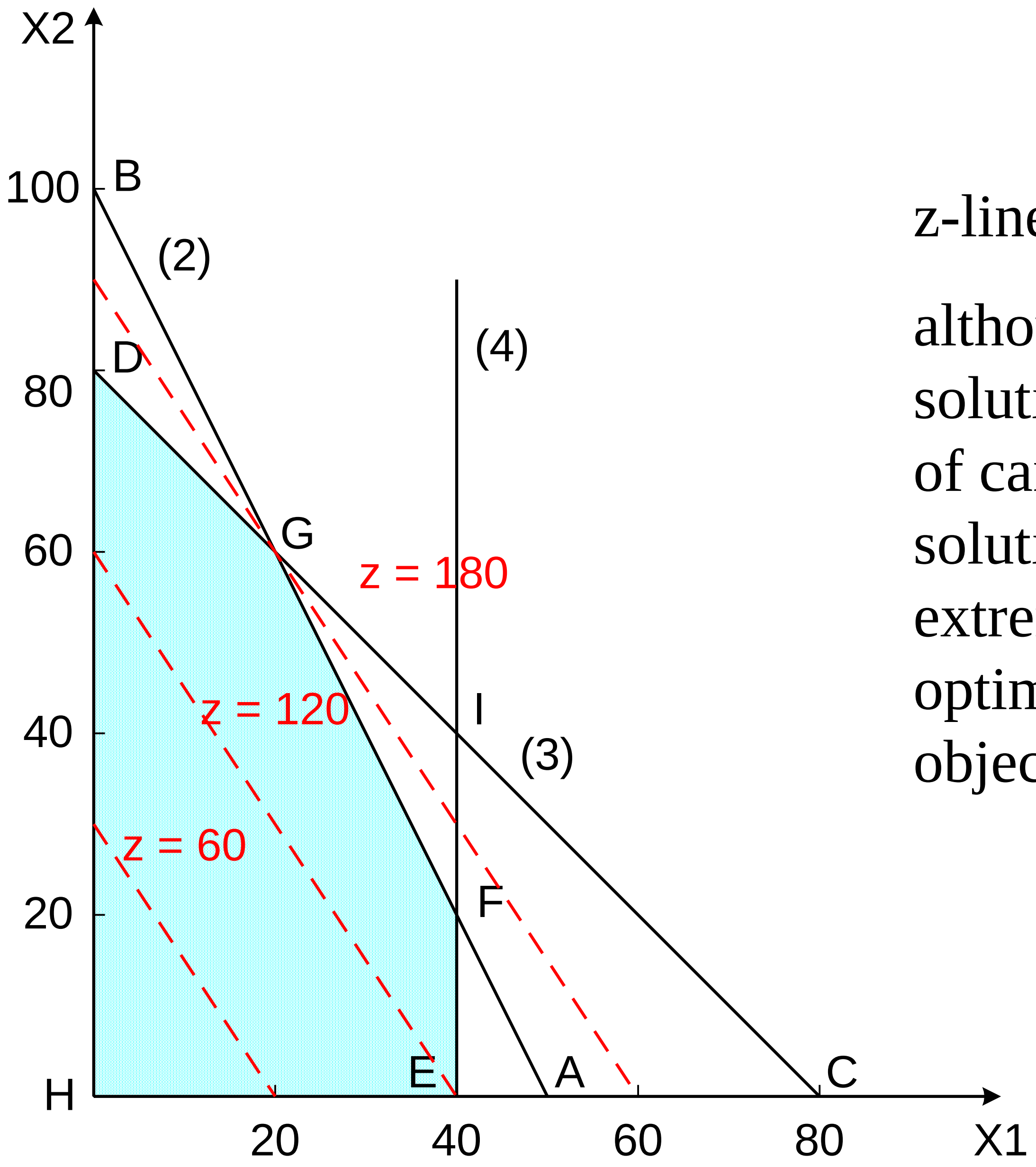

z-line = isoprofit-line

although the number of feasible solutions is infinite, the number of candidates for the optimal solutions is limited (i.e. corner or extreme points) → G is the optimal solution for this objective function

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Some definitions and concepts

- A constraint is **binding** when the available capacity is used for the optimal solution.
- Otherwise a constraint is **non-binding**.

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Applications

1. Dial-a-ride – TSP (travelling salesman problem) → patient transportation
2. BPP (bin packing problem) or VRP (vehicle routing problem) → planning of OR (operating rooms)

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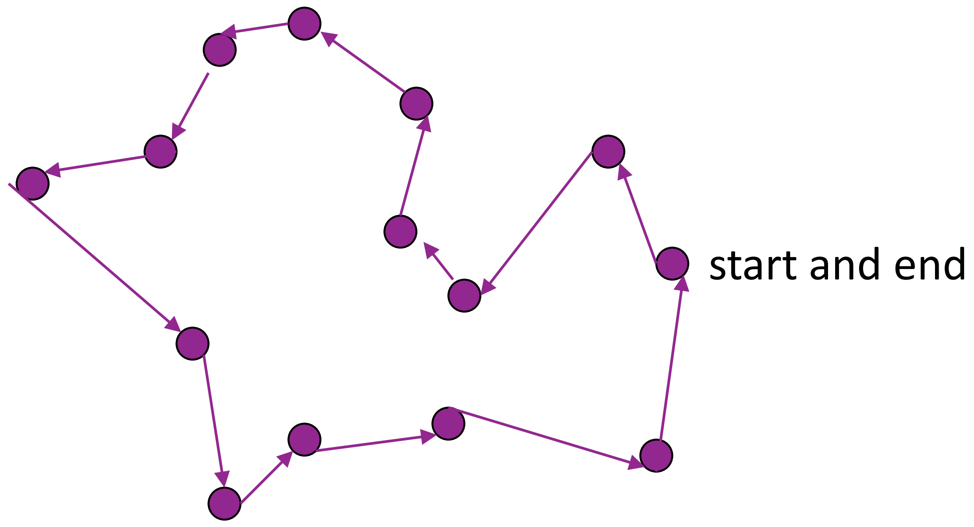
Application (1)

- Travelling salesman problem (basic problem)
  - A salesman must visit a number of cities starting from and returning to the same position. Determine the sequence to minimise the total distance travelled.
- Many variations and extensions exist: capacity limits, time windows, …



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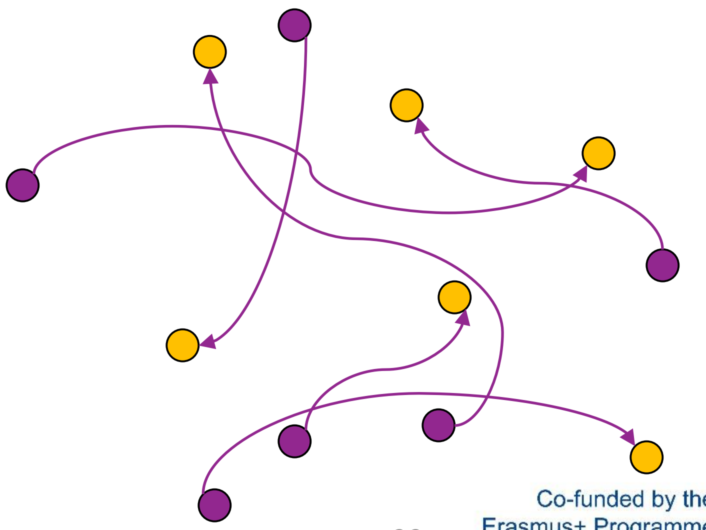
Application(1)

- Extensions are e.g. dial-a-ride problems
  - The *Dial-a-Ride Problem* (DARP) consists of designing vehicle routes and schedules for *n* users who specify pickup and delivery requests between origins and destinations. The aim is to plan a set of *m* minimum cost vehicle routes capable of accommodating as many users as possible, under a set of constraints.

origin: pickup

destination: delivery



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How to solve?

- If only one client at a time → (multiple) travelling salesman problem
  - moving from origin to destination using "shortest" path (fixed part)
  - connecting destination points with origin points (optimization): solution is assigning trips to different salesmen and sequencing the different demands for one salesmen

    use of adjusted distance matrix: destination – origin of trips

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Application(1)

- Applications within logistics: taxi, bus, …

patient transport in a hospital ? YES

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Application (1)

Dial-a-ride

- Patients:
  nursing ward → examination room
  and back

  - Transport mode = personel
  - Client/customer = patient
  - O/D origin/destination = nursing ward/RX

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Application (1)

Dial-a-ride

- Static – dynamic
  - Static: all movements known ahead, all durations of services are known
  - Dynamic: in reality good idea when forward moves have to be made, but returns? Duration of examination is not known in advance and so the sequence of orders can change, new orders arrive, transportation can go faster or slower (e.g. waiting time in front of elevator)
    → dynamic: real-time !!!

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Application (1)

How to solve?

- TSP is well known problems,

  but is extremely difficult to solve to optimality →
  use of (meta)heuristics: good solutions in a reasonable amount of time

- Many algorithms are available (free ware)
  - static: everything known in advance (sufficient time to calculate a good plan)
  - dynamic: use dispatching rules, regular updates

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Application (1)

- Other approaches are possible: e.g. CHU de Québec (*)
  - 14 employees dedicated to transportation
  - > 100,000 demands are carried out per year

  - different approach is used: selection of routes based on data analysis of origins and destinations, a pattern of routes is recognized based on transportation data of the previous year and people are assigned based on the frequency of these tours

(*) Ref.: Séguin, S., Villneuve, Y., Bloui-Delisle CH, Improving patient transportation in hospitals using a mixed-integer programming model, Operations Research for Health Care, 23 (2019), 100202.

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Application (2)

OR for OR

- Operations research and operating rooms: planning

- BBP ?
  VRP ?

How are these two problems linked?

Change over (cleaning and preparation) time between 2 operations: sequence independent or not?

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Application (2)

BBP: bin packing problem

Several items have to be packed in a number of bins with limited capacity. Try to put these items in the bin such that e.g. a maximum value is contained.

VRP: vehicle routing problem

A set of geographically dispersed customers with known requirements must be served by a fleet of vehicles stationed at a facility or depot in such a way as to minimize some distribution objective.

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BBP-approach

- Assuming that the preparation time of an operating room is independent of the previous operation and can be added to the operating time
- Link:
  - the bin = the operating room
  - the capacity of the bin = the available time in the operating room
  - size of an item = time required to perform the operation
  - objective: maximize number of items in the bin → number of operations
    maximize value of items in the bin → importance of operations

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BBP-approach

- In a first step items are assigned to bins (operations to operating rooms), then the second step defines the order based on secondary objectives (critical, unsure about operating time, possible complications, …)

- Filling up a set of bins is a well researched problem

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VRP-approach

- Assuming that the preparation time of an operating room is depending on the previous operation then this preparation time cannot be added to the operating time

- Link:
  - customer = operation
  - distance between customers (traveling time + service time) = preparation time + operating time
  - maximum tour length in time = available time in the operating room

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VRP-approach

- Similar procedure as with BBP
- In a first step assigning customers (=operations) to trucks (=operating rooms) but here neighbouring customers (= similar operations with small change over times) should be put together. In a second step the sequence chosen in such a way that total travelling time (change over time) is minimized (this comes down to solving a TSP).

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Conclusion

- Many problems from the area of healthcare can be solved using operations research and operations management techniques: e.g.: analysis of throughput time in an emergency unit can be linked to value analysis in combination with a SMED (single minute of dies) approach. But that is another story.

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